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Renewable Energy 00 (2014) 1–14

Renewable Energy

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A dimensional analysis for determining optimal discharge and penstock diameter in impulse and reaction water turbines

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Abstract

This paper presents a dimensional analysis for determining optimal flow discharge and optimal penstock diameter when designing impulse and reaction turbines for hydropower systems. The aim of this analysis is to provide general insights for minimizing water consumption when producing hydropower. This analysis is based on the geometric and hydraulic characteristics of the penstock, the total hydraulic head and the desired power production. As part of this analysis, various dimensionless relationships between power production, flow discharge and head losses were derived. These relationships were used to withdraw general insights on determining optimal flow discharge and optimal penstock diameter. For instance, it was found that for minimizing water consumption, the ratio of head loss to gross head should not exceed about 15%. Two examples of application are presented to illustrate the procedure for determining optimal flow discharge and optimal penstock diameter for impulse and reaction turbines.

Keywords:
Dimensional analysis, Hydropower, Impulse turbine, Optimal flow, Optimal penstock diameter, Reaction turbine

1. Introduction

The world energy consumption will grow by 56% between 2010 and 2040 [6]. As world population continues to grow and the limited amount of fossil fuels begins to diminish, there is an increasing demand to exploit renewable sources of energy.

In the United States, about 9% of all energy consumed in 2012 was from renewable sources [7]. While this is a relatively small fraction of the U.S. energy supply in 2012, the United States was the world's largest consumer of renewable energy from geothermal, solar, wood, wind, and waste for electric power generation producing almost 25% of the world's total [7]. This institute also reports that in 2012, 30% of the renewable energy in the U.S. was from hydropower. This means that only about 3% of all energy consumed in the United States was from hydropower.

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Globally, hydropower accounted for 16% of all global electricity production in 2007, with other renewable energy sources totalling 3% [5]. Hence, it is not surprising that when options are evaluated for new energy developments, there is strong impulse for fossil fuel or nuclear energy as opposed to renewable sources. However, as hydropower schemes are often part of a multipurpose water resources development project, they can often help to finance other important functions of the project [3]. In addition, hydropower provides benefits that are rarely found in other sources of energy. In fact, dams built for hydropower schemes, and their associated reservoirs, provide human well-being benefits, such as securing water supply, flood control and irrigation for food production, and societal benefits such as increased recreational activities and improved navigation [3].

Furthermore, hydropower due to its associated reservoir storage, can provide flexibility and reliability for energy production in integrated energy systems. The storage capability of hydropower systems can be seen as a regulating mechanism by which other diffuse and variable renewable energy sources (wind, wave, solar) can play a larger role in providing electric power of commercial quality [5]. While development of all the remaining hydroelectric potential could not hope to cover total future world demand for electricity, implementation of the remaining potential can make a vast contribution to improving living standards in the developing world (South America, Asia and Africa), where the greatest potential still exists [7].

Minimizing water consumption for producing hydropower is critical given that overuse of flows for energy production may result in a shortage of flows for other purposes such as irrigation or navigation. The present work was motivated when the first author was unable to find in the literature a theoretical framework for determining optimal flow discharge and optimal penstock diameter for the design of impulse and reaction turbines. Recently, Pelz [4] provided a theoretical approach for determining the upper limit for hydropower gained by a water wheel or turbine per unit width in a rectangular open-channel. This is somewhat different of impulse and reaction turbines, as in the latter turbines, the flow in the penstock is pressurized.

This paper aims to provide general insights on determining optimal flows and optimal penstock diameters when designing impulse and reaction turbines for hydropower systems. This paper is divided as follows. First, dimensionless relationships between power production, flow discharge and head losses are derived. Second, these relationships are used to withdraw general insights on determining optimal flow discharge and optimal penstock diameter. Third, examples of application for determining optimal flows when designing impulse and reaction turbines are presented. Finally, the key results are summarized in the conclusion.

2. Dimensional analysis for optimal flow discharge, optimal head losses and optimal power

The electric power, P, in Watts (W), can be determined by the following equation:

$$P = \eta \gamma Q (H_g - h_L) \tag{1}$$

where γ (= $\rho \times g$) is specific weight of water in kg/(m^2 × s^2), Q is flow discharge in m^3/s, H_g is gross head in m, h_L is sum of head losses in m, ρ is water density in kg/m^3, g is acceleration of gravity in m/s^2, and η is overall hydroelectric unit efficiency, which in turn is the product of turbine efficiency (η_t) and generator efficiency(η_g). In all derivations presented in this paper, it is assumed that η (= $\eta_t \times \eta_g$) is constant.

For an impulse turbine (see Fig. 1), the sum of head losses can be written as

$$h_L = \frac{Q^2}{2gA_2^2}\left[f\frac{L}{D_2} + \sum k_{1-2} + k_N \left(\frac{A_2}{A_N}\right)^2 \right] \tag{2}$$

where L, D_2 and A_2 are length, diameter and cross-sectional area of penstock, respectively. In addition, f is friction factor, $\sum k_{1-2}$ is the sum of local losses in penstock due to entrance, bends, penstock fittings and gates, A_N is nozzle area at its exit (section 3 in Fig. 1) and k_N is nozzle head loss coefficient, which is given by (e.g., [1]).

$$k_N = \frac{1}{C_V^2} - 1 \tag{3}$$

where C_V is nozzle velocity coefficient. According to Dixon (2005), C_V varies between 0.98 and 0.99 for a typical Pelton turbine nozzle.

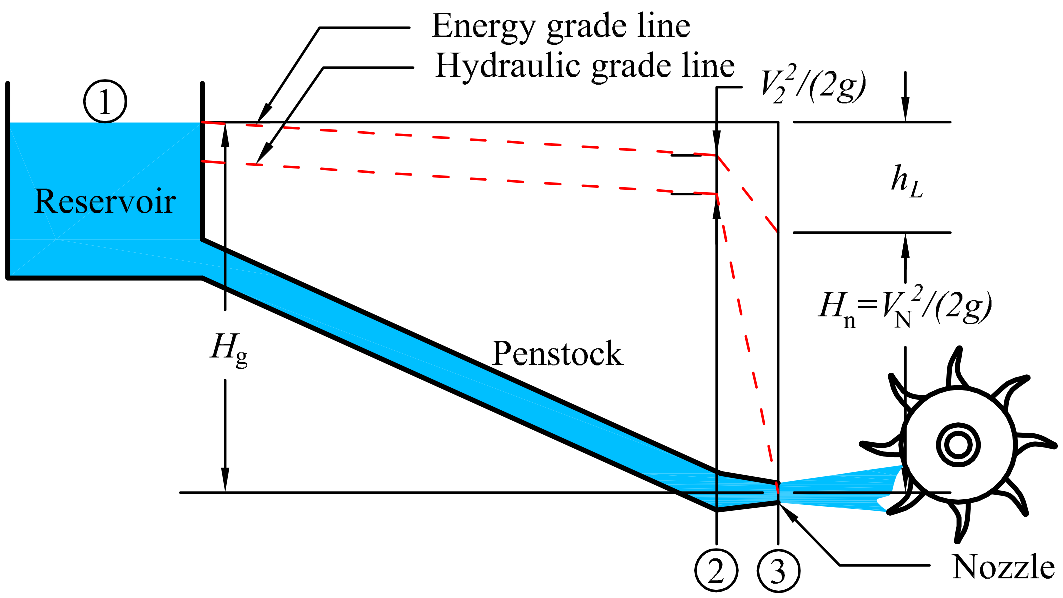

Fig. 1. Sketch of an impulse turbine

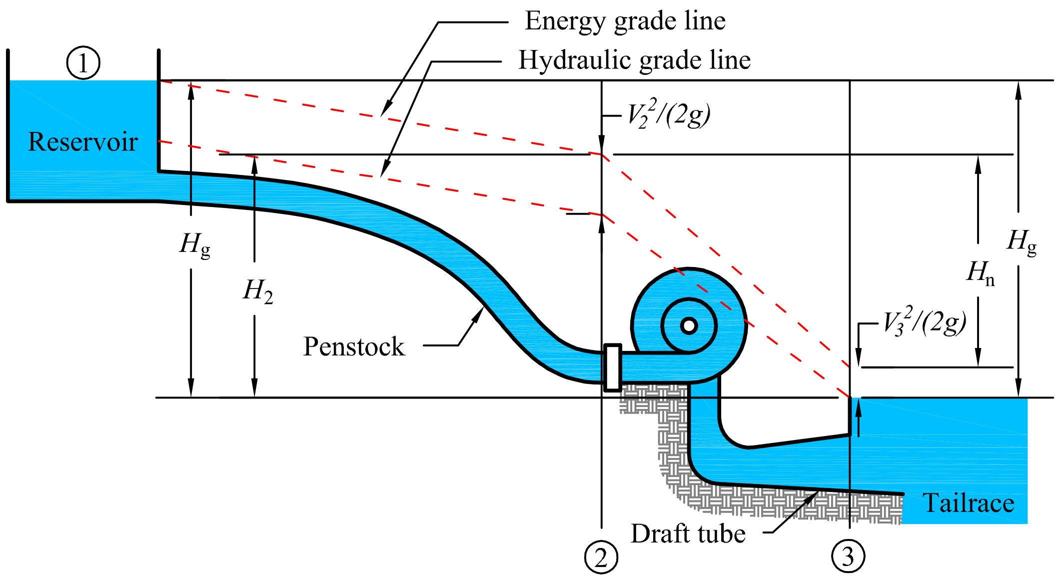

Fig. 2. Sketch of a reaction turbine

For a reaction turbine (see Fig. 2), the sum of head losses can be written as

$$h_L = \frac{Q^2}{2gA_2^2}\left[f\frac{L}{D_2} + \sum k_{1-2} + \left(\frac{A_2}{A_d}\right)^2\right] \tag{4}$$

where A_d is draft tube cross-sectional area at its outlet (section 3 in Fig. 2).

The expression inside the brackets in Eqs. (2) and (4) is dimensionless and it is denoted herein as

$$C_L = \begin{cases} f\frac{L}{D_2} + \sum k_{1-2} + k_N\left(\frac{A_2}{A_N}\right)^2 & \text{for an impulse turbine} \\ f\frac{L}{D_2} + \sum k_{1-2} + \left(\frac{A_2}{A_d}\right)^2 & \text{for a reaction turbine} \end{cases} \tag{5}$$

Hence, the total head losses in Eq. (2) and Eq. (4) is equal to the product of C_L and $Q^2/(2gA_2^2)$ and thus, Eq. (1) can be written as

$$P = \eta\gamma Q(H_g - C_L\frac{Q^2}{2gA_2^2}) \tag{6}$$

For generalizing the findings in this paper, a dimensionless relationship between power and flow discharge is sought. To achieve this, Eq. (6) is divided by a reference power (P_r). P_r is assumed to be the maximum power that can be generated using a reference discharge (Q_r) and a fixed gross head and penstock geometry (constant C_L). For maximum power, the turbine and generator efficiencies need to be 100% (i.e., η_t = 100% and η_g = 100%). Also, maximum power for a fixed penstock geometry can be obtained by setting dP/dQ in Eq. (6) equal to zero, which gives

$$h_L = \frac{H_g}{3} \tag{7}$$

The reference flow discharge Q_r can be obtained by using Eq. (7) and the energy equation between the reservoir and the nozzle exit for an impulse turbine or between the reservoir and the tailrace for a reaction turbine, which gives:

$$Q_r = 2A_3\sqrt{\frac{1}{3}gH_g} \tag{8}$$

where A_3 is the cross-sectional area at section 3 in Figs. 1 and 2, given by

$$A_3 = \begin{cases} A_N & \text{for an impulse turbine} \\ A_d & \text{for a reaction turbine} \end{cases} \tag{9}$$

Substituting Eq. (7) and Eq. (8) into Eq. (1) gives the following relation for the reference power (P_r)

$$P_r = \frac{4}{3}\gamma H_g A_3\sqrt{\frac{1}{3}gH_g} \tag{10}$$

Note that Q_r and P_r (Eqs. 8 and 10) are a function of the penstock properties and the gross head only. Dividing each side of Eq. (6) by P_r (Eq. 10) and defining P/P_r as P_+ and Q/Q_r as Q_+, and after some algebra, the following dimensionless relationship between power and discharge is obtained

$$P_+ = \eta\left[\frac{3}{2}Q_+ - C_L\left(\frac{A_3}{A_2}\right)^2 Q_+^3\right] \tag{11}$$

Denoting with β the product of C_L and $(A_3/A_2)^2$, Eq. (11) can be rewritten as

$$P_+ = \eta\left(\frac{3}{2}Q_+ - \beta Q_+^3\right) \tag{12}$$

where

$$\beta = \begin{cases} \left(\frac{A_N}{A_2}\right)^2\left(f\frac{L}{D_2} + \sum k_{1-2} + k_N\left(\frac{A_2}{A_N}\right)^2\right) & \text{for an impulse turbine} \\ \left(\frac{A_d}{A_2}\right)^2\left(f\frac{L}{D_2} + \sum k_{1-2} + \left(\frac{A_2}{A_d}\right)^2\right) & \text{for a reaction turbine} \end{cases} \tag{13}$$

In practice, the ratios A_N/A_2 and A_d/A_2 in Eq. (13) are typically kept constant, which means that β varies as a function of f, L, D_2, and the coefficients of local head losses ($\sum k$). In many applications, friction losses are more important than local head losses, that is $f\frac{L}{D_2} \gg \sum k$. Also, L is typically constant as it is restricted by topographic conditions. In addition, f does not show significant variation as a function of discharge or penstock diameter. Let's recall that for a given penstock diameter, f is independent of the Reynolds number for fully developed turbulent flows, which is the case of most penstock flows. Hence, β is more or less inversely proportional to the penstock diameter, $D_2 \propto 1/\beta$.

The variation of P_+ with respect to Q_+ for a fixed β can be obtained by differentiating P_+ with respect to Q_+ in Eq. (12), which gives

$$\frac{dP_+}{dQ_+} = \eta\left(\frac{3}{2} - 3\beta Q_+^2\right) \tag{14}$$

The maximum dimensionless power for a fixed β can be obtained by setting dP_+/dQ_+ in Eq. (14) equal to zero. The maximum power occurs when

$$(Q_+)_{\max} = \sqrt{\frac{1}{2\beta}} \tag{15}$$

The maximum dimensionless power for a fixed β is obtained by substituting Q_+ from Eq. (15) in Eq. (12), which gives

$$(P_+)_{\max} = \eta\sqrt{\frac{1}{2\beta}} \tag{16}$$

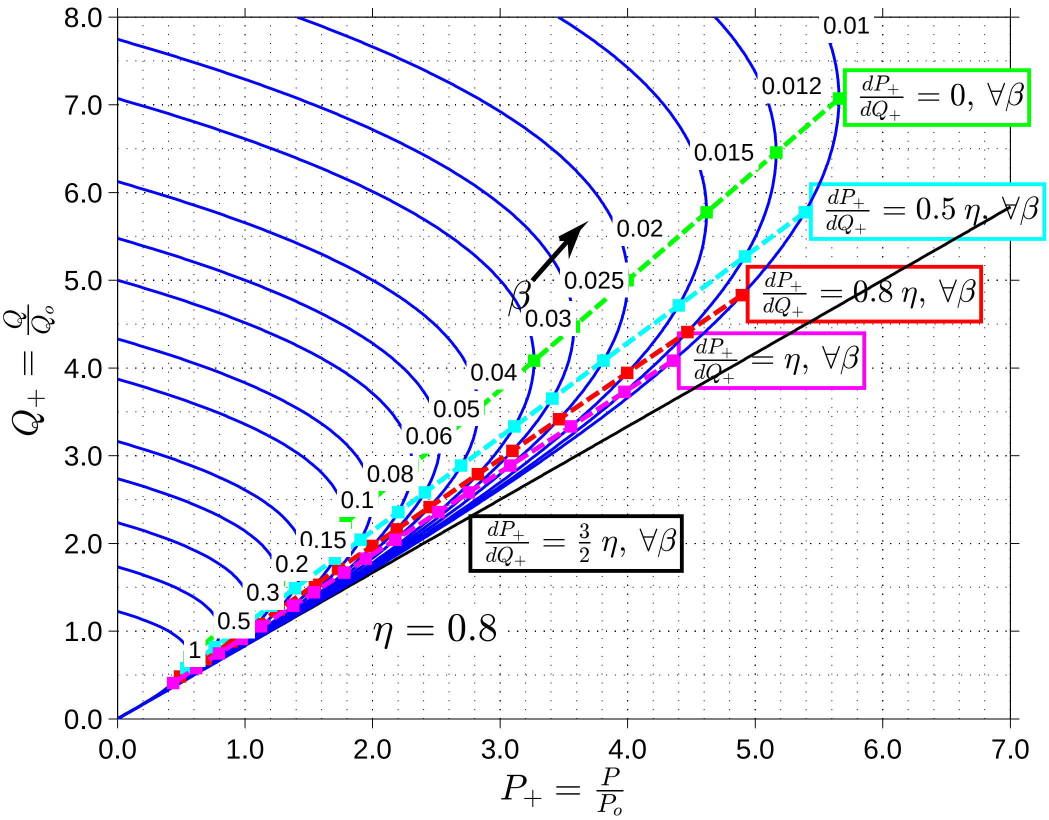

Fig. 3. Dimensionless discharge (Q_+) versus dimensionless power (P_+) for $\eta = 0.8$ and a typical range of β for impulse turbines

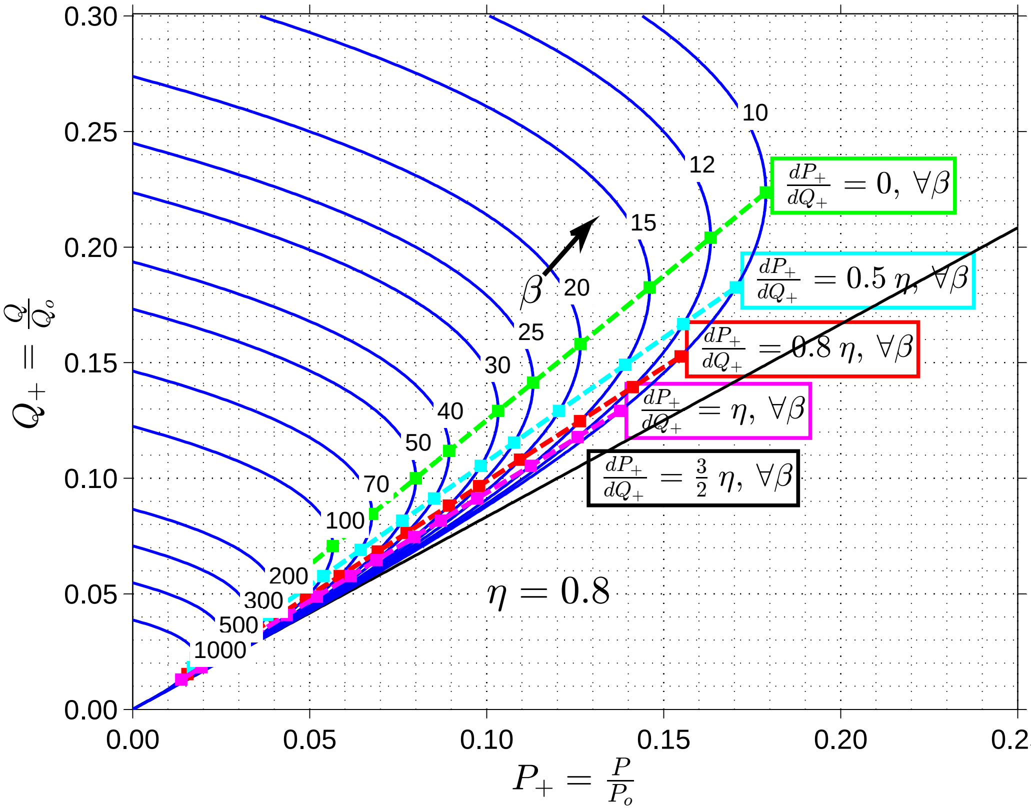

Fig. 4. Dimensionless discharge (Q_+) versus dimensionless power (P_+) for $\eta = 0.8$ and a typical range of β for reaction turbines

In most applications, β should range between 0.01 and 1.0 for impulse turbines, and between 10 and 1000 for reaction turbines. Likewise, C_L should range between 1 and 100 for both, impulse and reaction turbines. Even though β is used throughout the entire paper, only C_L is needed for design purposes.

Figures (3) and (4) plot Q_+ versus P_+ in Eq. (12) for typical ranges of β for impulse and reaction turbines, respectively. An overall hydroelectric unit efficiency (η) of 0.8 was used for plotting these figures. As can be observed in Figs. 3 and 4, the change in power production in relation to change in flow discharge ($\Delta P_+/\Delta Q_+$) for each dimensionless curve has a positive and negative gradient. For optimizing power production, only the positive gradient is of interest ($\Delta P_+/\Delta Q_+ > 0$).

To visualize changes in power production in relation to changes in flow discharge, five ratios of dP_+/dQ_+ in Eq. (14) are plotted in Figs. 3 and 4. Note in Figs. 3 and 4 that for a given β, the positive range of dP_+/dQ_+ varies from $(3/2)\eta$ to 0. Note also that dP_+/dQ_+ changes rapidly near $(Q_+)_{\max}$ and, that in the positive range of dP_+/dQ_+, the maximum relative power P_+ occurs for the maximum relative flow discharge Q_+.

For minimizing water consumption to produce a given amount of hydropower, it is necessary that dP_+/dQ_+ in Eq. (14) is close to its maximum value $(3/2)\eta$. Note in Figs. 3 and 4 that for each curve between approximately $dP_+/dQ_+ = (3/2)\eta$ and $dP_+/dQ_+ = 0.8\eta$, the increase in dimensionless power (P_+) is approximately linear with increase in dimensionless discharge (Q_+). Note also in these Figures that for dP_+/dQ_+ smaller than about 0.8η, the increase in P_+ is small compared to the increase in Q_+. Herein, to minimize water consumption, the optimal lower limit of dP_+/dQ_+ is set to 0.8η.

Substituting $dP_+/dQ_+ = 0.8\eta$ into Eq. (14) gives the following upper limit for the dimensionless flow discharge,

$$(Q_+)_{\text{opt upper}} = \sqrt{\frac{7}{30\beta}} \tag{17}$$

The corresponding upper limit for the dimensionless power is

$$(P_+)_{\text{opt upper}} = \eta\frac{19}{15}\sqrt{\frac{7}{30\beta}} \tag{18}$$

Hence, the optimal dimensionless discharge range is $Q_+ \in [0, \sqrt{7/(30\beta)}]$. The corresponding optimal dimensionless power range is $P_+ \in [0, \eta\frac{19}{15}\sqrt{\frac{7}{30\beta}}]$.

The optimal dimensionless head loss ($h_{L+} = h_L/H_g$) can be obtained by assuming that the optimal upper limit for the flow discharge is $Q_+ = \sqrt{7/(30\beta)}$ (Eq. 17). In Eq. (12), dividing the second term of the right-hand side (RHS) by the first term of the RHS gives

$$h_{L+} \leq \frac{2}{3}\beta Q_+^2 \tag{19}$$

Substituting $(Q_+)_{\text{opt upper}} = \sqrt{7/(30\beta)}$ into Eq. (19) gives

$$h_{L+} \leq \frac{7}{45} \tag{20}$$

Eq. (20) shows that for minimizing water consumption, the ratio of head loss to gross head ($h_{L+} = h_L/H_g$) should not exceed 15.6%. The 15.6% ratio also provides the threshold for the optimal penstock diameter. Losses higher than 15.6% mean that a small penstock diameter is used. The 15.6% ratio is about half of that derived for maximum power and maximum flow discharge, which is 33.3%. This means that the optimal conditions for producing power do not correspond to those that use maximum flow discharge for a given β. This can be better understood by observing Figs. 3 and 4, in which dP_+/dQ_+ decreases rapidly near $(P_+)_{\max}$ for all β.

So far the analysis assumed that β is constant and hence, the penstock diameter (D_2). Following, the influence of changing the penstock diameter on power production is assessed. Earlier, it was argued that D_2 and β are more or less inversely proportional. For example, reducing β in half is approximately equivalent to doubling the penstock diameter. An increase in penstock diameter in turn results in a decrease in head losses and hence, an increase in power. For estimating the variation of P_+ with respect to β ($\Delta P_+/\Delta\beta$), Q_+ in Eq. (12) is kept constant, which gives

$$\frac{\Delta P_+}{\Delta\beta} = -\eta Q_+^3 \tag{21}$$

By combining Eqs. (12) and (21), and after some algebra, the following relationship between $\Delta P_+/P_+$ and $\Delta\beta/\beta$ is obtained

$$\frac{\Delta P_+}{P_+} = -\frac{\Delta\beta}{\beta}\left(\frac{\beta Q_+^3}{\frac{3}{2}Q_+ - \beta Q_+^3}\right) \tag{22}$$

Note in Eq. (22) that the maximum relative power increase $(\Delta P_+/P_+)_{\max\ \text{incr}}$ will occur when $\Delta\beta/\beta$ = -1, which would take place in the hypothetical case that β is reduced to zero. The relationship between β and Q_+ when $dP_+/dQ_+ = 0$ can be obtained from Eq. (15), which gives $\beta = 1/(2(Q_+)^2_{\max})$. By substituting this β into Eq. (22), and using $\Delta\beta/\beta$ = -1, gives $(\Delta P_+/P_+)_{\max\ \text{incr}} = 1/2$. Likewise an increase in β will result in a decrease in power. It should be noted that the maximum relative power decrease $(\Delta P_+/P_+)_{\max\ \text{decr}}$ is $\Delta P_+/P_+$ = -1, which would occur in the case that P_+ is reduced to zero.

Figs. 5 and 6 plot the variation of $\Delta P_+/P_+$ versus $\Delta\beta/\beta$ for two different values of Q_+ (i.e., 0.06 and 3).

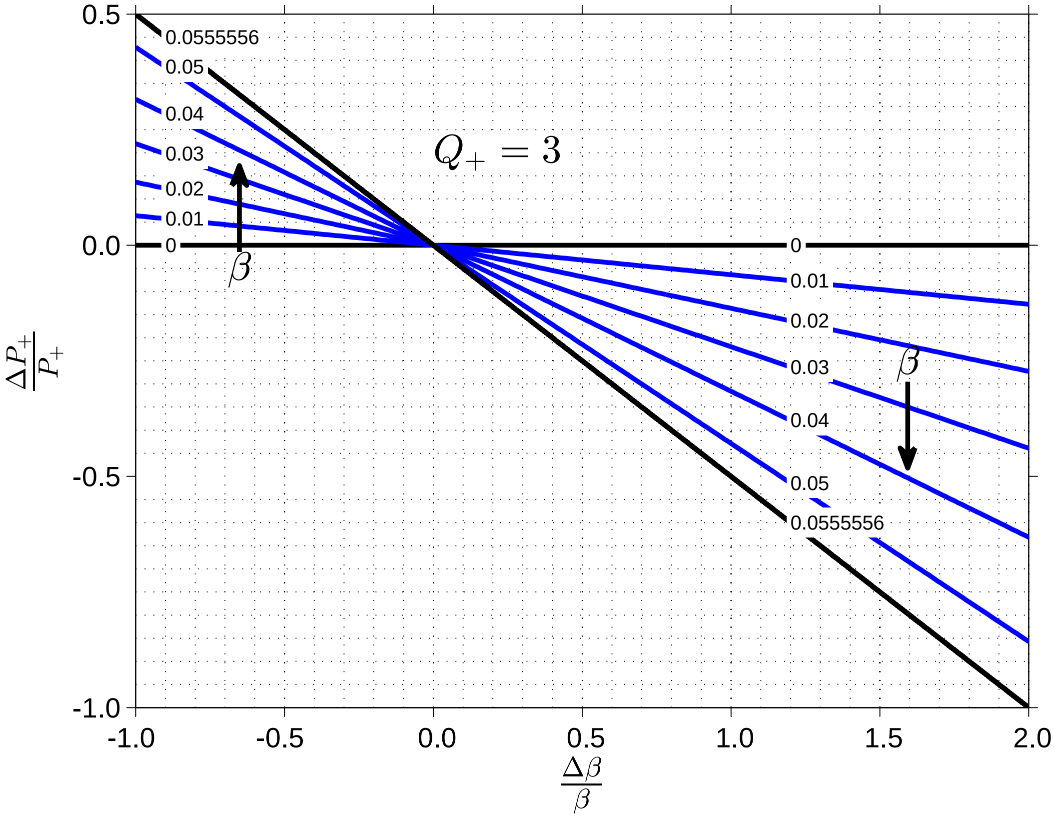

Fig. 5. $\Delta P_+/P_+$ versus $\Delta\beta/\beta$ for $Q_+ = 3$

The larger value of Q_+ (i.e., $Q_+ = 3$) is typical of an impulse turbine, while the smaller value (i.e., $Q_+ = 0.06$) corresponds to that of a reaction turbine. Note in Figs. 5 and 6 that relative power is increased when β is reduced and viceversa.

For the assumed optimal flow conditions (see Eqs. 17 - 20), the maximum relative power increase can be obtained by substituting $(Q_+)_{\text{opt upper}} = \sqrt{7/(30\beta)}$ (Eq. 17) and $\Delta\beta/\beta = -1$ into Eq. (22), which gives $\Delta P_+/P_+ = 18.4\%$. If β is reduced in half (D_2 is approximately doubled), $\Delta P_+/P_+ = 9.2\%$. In other words, for the assumed optimal flow conditions, a gain of about 9% in power production can be attained by doubling the penstock diameter.

For practical applications, the derived dimensionless relationships are made non-dimensionless. For instance, the optimal upper limit of the flow discharge can be obtained by combining Eqs. (8) and (17), which after some algebra gives

$$Q_{\text{opt}} = \frac{2}{3} A_2 \sqrt{\frac{7}{10}\frac{gH_g}{C_L}} \tag{23}$$

Similarly, the optimal upper limit of the power can be obtained by combining Eqs. (10) and (18), which after some algebra gives

$$P_{\text{opt}} = \frac{76}{135}\eta\gamma H_g A_2 \sqrt{\frac{7}{10}\frac{gH_g}{C_L}} \tag{24}$$

When designing a turbine, it is necessary to specify either the flow discharge to use or the desired electric power. These cases are presented below:

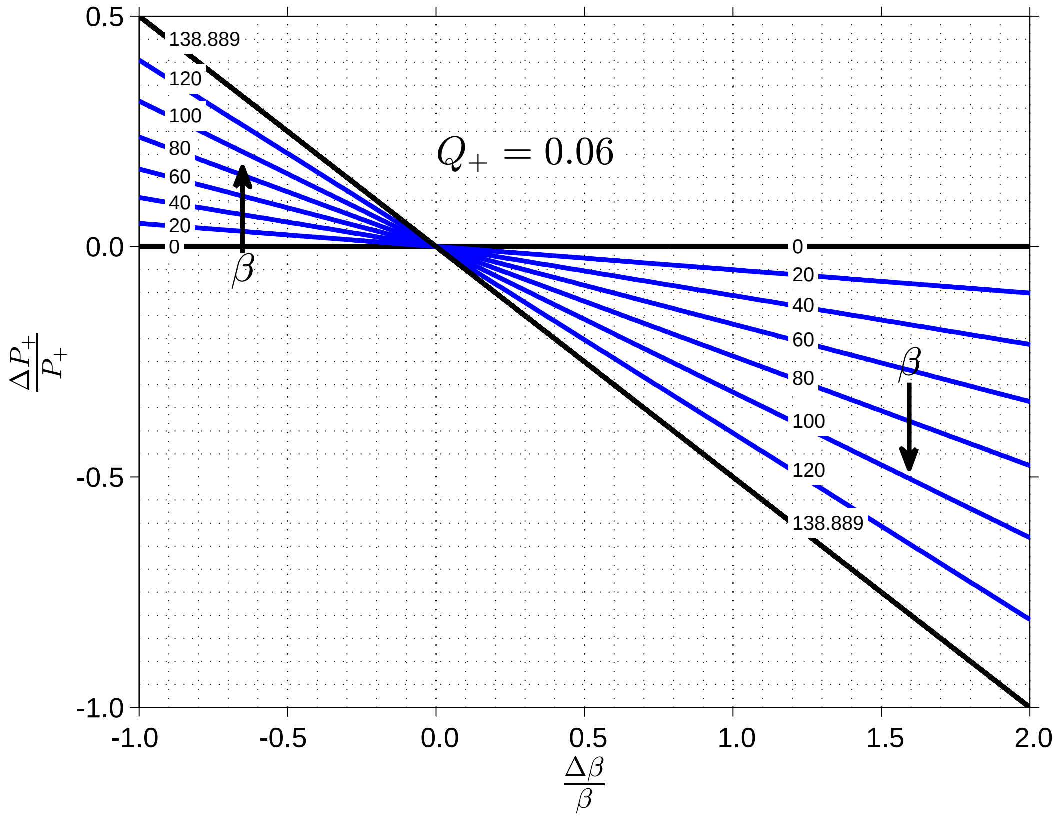

Fig. 6. $\Delta P_+/P_+$ versus $\Delta\beta/\beta$ for $Q_+ = 0.06$

2.1. P is specified

If P is specified, the optimal upper limit of the flow discharge can be obtained by combining Eqs. (23) and (24), which gives

$$Q_{\text{opt}} = \frac{45}{38}\left(\frac{P}{\eta\gamma H_g}\right) \tag{25}$$

The optimal penstock diameter can be determined from Eq. (23) as follows

$$\frac{(C_L)_{\text{opt}}}{A_2^2} \leq \frac{14}{45}\frac{gH_g}{Q^2} \tag{26}$$

where Q in Eq. (26) is the same as that in Eq. (25).

2.2. Q is specified

If Q is specified, the optimal upper limit of the power can be obtained by combining Eqs. (23) and (24), which gives

$$P_{\text{opt}} = \frac{38}{45}\eta\gamma H_g Q \tag{27}$$

In this case, the optimal penstock diameter can still be determined using Eq. (26).

It is pointed out that the proposed methodology for determining the optimal flow discharge and optimal penstock diameter does not account for cavitation. Reaction turbines (not impulse turbines) are subjected to cavitation. In reaction turbines, cavitation may occur at the outlet of the runner or at the inlet of the draft tube where the pressure is considerably reduced (Dixon 2005). In order to determine whether cavitation will occur in any portion of a reaction turbine, the Thoma's cavitation factor (σ) is compared with the critical cavitation factor (σ_c). If the value of σ is greater than σ_c cavitation will not occur in the turbine under analysis, where σ_c is a function of the specific speed of the turbine (N_s). Because N_s is not used in the proposed methodology, the occurrence of cavitation cannot be determined using the utilized parameters. The occurrence of cavitation in reaction turbines needs be checked after using the proposed methodology.

Following two examples of application for determining optimal flow discharge and optimal penstock diameter for an impulse turbine and a reaction turbine are presented.

3. Example of application for an impulse turbine

The site, penstock and nozzle characteristics for this example are as follows:

1. Gross head (H_g) = 200 m
2. Penstock length (L) = 500 m
3. Ratio of penstock cross-sectional area to nozzle cross-sectional area at its outlet (A_2/A_N) = 16
4. Nozzle velocity coefficient (C_V) = 0.985
5. Sum of local losses in penstock due to entrance, bends, penstock fittings and gates ($\sum k_{1-2}$) = 1.5
6. Roughness height of penstock material (ϵ) = 0.045 mm (commercial steel)
7. Kinematic Viscosity (ν) = 10^{-6} m^2/s
8. Turbine efficiency (η_t) = 82%
9. Generator efficiency (η_g) = 90%

3.1. Case A1: Q is specified

In this case, let's assume that the design flow Q is 0.6 m^3/s and it is desired to know the optimal hydropower that can be extracted using this flow. First, it is necessary to determine the optimal penstock diameter. From Eq. (26),

$$\frac{(C_L)_{\mathrm{opt}}}{A_2^2} = 1693.8272\ \mathrm{m}^{-4} \tag{28}$$

where $C_L = 500f/D_2 + 1.5 + k_N(16^2)$.

The nozzle coefficient is determined using Eq. (3), which gives $k_N = 0.0307$. The friction factor (f) is determined using the explicit Swamee−Jain equation which is given by

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon}{3.7D_2} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \tag{29}$$

where ϵ is the roughness height and Re is the Reynolds number. The Reynolds number is defined as VD_2/ν, where V is the flow velocity. Note that when Q is known, f and C_L are functions of D_2 only. Solving for D_2 in the above relation of $(C_L)_{\mathrm{opt}}/(A_2^2)$ gives $D_2 = 0.3968$ m. In practice, a penstock with an internal diameter equal or slightly larger than 0.3968 m (397 mm) would be selected. Assuming that a schedule 80 steel pipe is required due to structural considerations, a 18 in outside diameter pipe would be selected. For this pipe, the wall thickness is 0.938 in, and hence the internal diameter is 16.124 in (409.5 mm). For this pipe diameter, the value of C_L is 25.35. This value can be used to determine the dimensionless head loss as follows (e.g., second and first terms in Eq. (6), respectively).

$$h_{L+} = C_L\frac{Q^2}{2gH_gA_2^2} = 0.134 \text{ or } 13.4\% \tag{30}$$

which satisfies the inequality in Eq. (20) [< 15.6%].

The electric power that can be extracted from this system can be determined using Eq. (6), which gives,

$$P = 0.82 \times 0.90 \times 1000 \times 9.8 \times 0.6 \times \left(200 - 25.35 \times \frac{0.6^2}{2 \times 9.8 \times 0.1317^2}\right) = 751421 \text{ W } = 751.4 \text{ kW} \quad (31)$$

To facilitate the calculations, a Matlab hydropower calculator was developed which Graphical User Interface (GUI) is shown in Fig. 7. As can be observed in this Figure, the consumption of flow is optimized in the linear region because the amount of power is proportional to the amount of flow used. Right before the the high positive gradient in each curve, both the flow discharge and the penstock diameter are optimized. The hydropower calculator is available at `http://web.engr.oregonstate.edu/~leon/Codes/Hydropower/`.

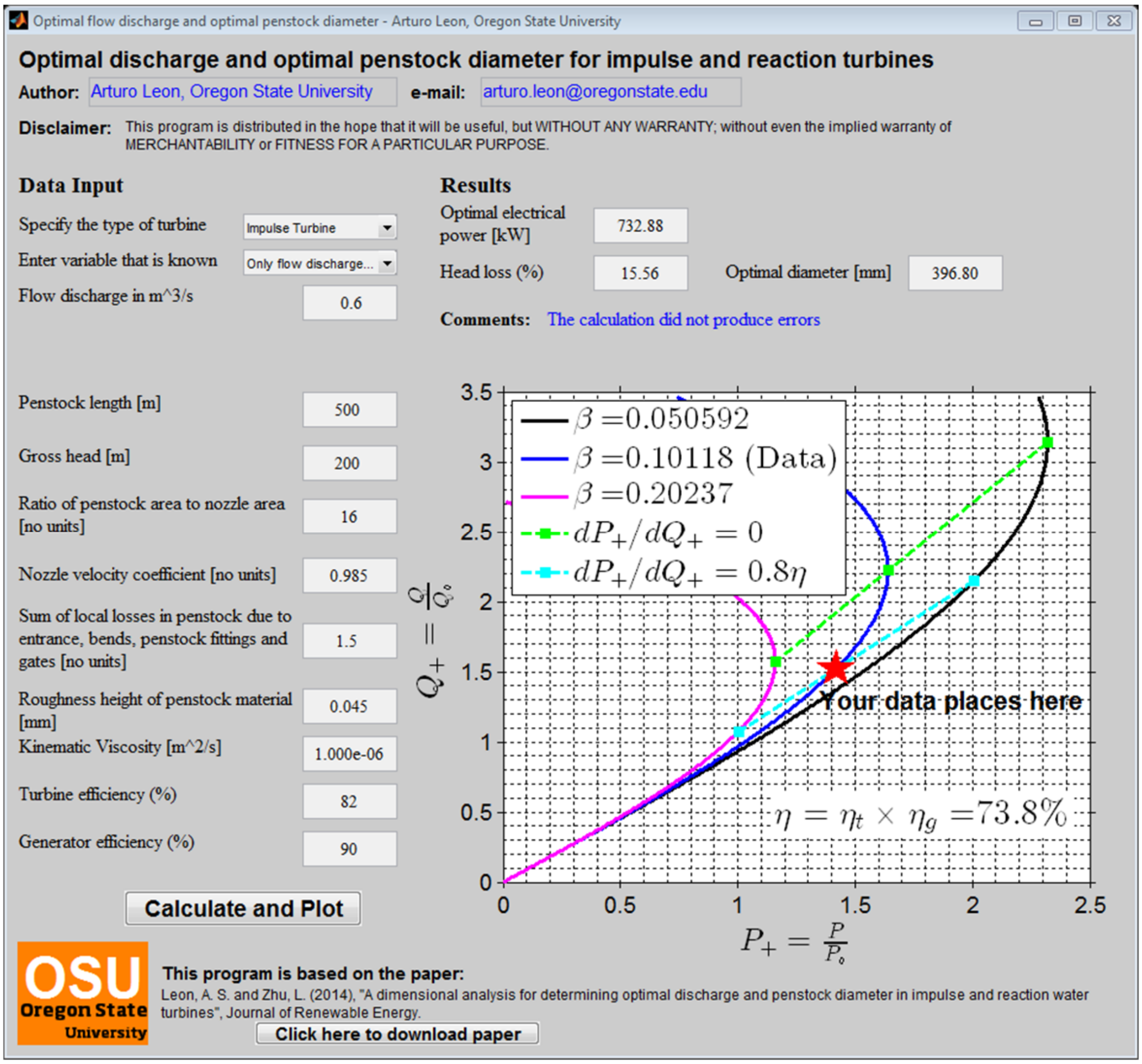

Fig. 7. Graphical User Interface (GUI) of hydropower calculator

3.2. Case B1: P is specified

In this case, assume that P is 100 kW and it is desired to determine the optimal flow discharge and optimal penstock diameter to produce this power. In this case, first, the optimal discharge is determined using Eq. (25) as follows:

$$Q_{\text{opt}} = \frac{45}{38}\left(\frac{100,000}{0.82 \times 0.90 \times 1000 \times 9.8 \times 200}\right) = 0.082\ \text{m}^3/\text{s}\ (82\ \text{L/s}) \tag{32}$$

The optimal pipe diameter (inside diameter) can de determined in a similar way to *Case A1*, which gives 0.176 m.

4. Example of application for a reaction turbine

The site and penstock characteristics for this example are the same as those of the impulse turbine example. A new parameter for this example is

1. Ratio of penstock cross-sectional area to draft tube cross-sectional area at its outlet $(A_2/A_d) = 1/3$

4.1. Case A2: Q is specified

As in the case of an impulse turbine, assume that the design flow Q is 0.6 m^3/s and it is desired to determine the optimal penstock diameter, and the optimal electric power that can be extracted using this flow. First, it is necessary to determine the optimal penstock diameter. From Eq. (26),

$$\frac{(C_L)_{\text{opt}}}{A_2^2} = 1693.8272\ \text{m}^{-4} \tag{33}$$

where $C_L = 500f/D_2 + 1.5 + (1/3)^2$.

In a similar manner to case A1, solving for D_2 in the above relation of $(C_L)_{\text{opt}}/(A_2^2)$ gives $D_2 = 0.3696$ m. Assuming again that a schedule 80 steel pipe is required due to structural considerations, a 18 in outside diameter pipe would be selected. For this pipe, the wall thickness is 0.938 in, and hence the internal diameter is 16.124 in (409.5 mm). For this pipe diameter, the value of C_L is 17.60. Again, this value can be used to determine the dimensionless head loss, which gives 9.3%. This dimensionless head loss satisfies the inequality in Eq. (20) (< 15.6%). The electric power that can be extracted from this system can be determined using Eq. (6), which gives 787.01 kW, which in turn is slightly larger than that determined using an impulse turbine and the same flow discharge.

4.2. Case B2: P is specified

As in the case of an impulse turbine, assume that P is 100 kW and it is desired to determine the optimal flow discharge and optimal penstock diameter to produce this power. In this case, the optimal discharge is determined using Eq. (25), which gives 82 L/s. After the optimal flow discharge has been determined, a similar procedure to *Case A2* can be followed to determine the optimal pipe inside diameter. The optimal pipe inside diameter results in 0.171 m. This diameter is slightly smaller than that found for an impulse turbine and for the same flow discharge.

5. Conclusions

This paper presents a dimensional analysis for determining optimal flow discharge and optimal penstock diameter when designing impulse and reaction turbines for hydropower systems. The aim of this analysis is to provide general insights for minimizing water consumption when producing hydropower. The key findings are as follows:

1. The analysis is based on the geometric and hydraulic characteristics of the penstock, the total hydraulic head, and the desired power production.

2. This analysis resulted in various dimensionless relationships between power production, flow discharge and head losses.
3. The derived relationships were used to withdraw general insights on determining optimal flow discharge and optimal penstock diameter. For instance, it was found that for minimizing water consumption, the ratio of head loss to gross head (h_L/H_g) should not exceed about 15%.
4. To facilitate the calculations, a Matlab hydropower calculator was developed which is available at `http://web.engr.oregonstate.edu/~leon/Codes/Hydropower/`.
5. Overall, the present analysis is general and can be used for determining optimal design flow and penstock diameter when designing impulse and reaction turbines.

Acknowledgments

The authors gratefully acknowledge the financial support of the Bonneville Power Administration of the U.S. Department of Energy (DOE) under award number TIP#258.

Notation

The following symbols are used in this paper:

A_2	=	penstock cross-sectional area;
A_d	=	draft tube cross-sectional area at its outlet;
A_N	=	nozzle cross-sectional area;
C_V	=	Nozzle velocity coefficient;
C_L	=	dimensionless parameter that is function of penstock properties only;
D_2	=	penstock diameter;
g	=	acceleration due to gravity;
H_g	=	gross head;
h_L	=	sum of head losses;
k_N	=	nozzle head loss coefficient;
L	=	Penstock length;
P_r	=	reference power;
P_+	=	P/P_r;
Q	=	flow discharge;
Q_r	=	reference flow discharge;
Q_+	=	Q/Q_r;
β	=	product of C_L and $(A_3/A_2)^2$;
ϵ	=	roughness height;
η	=	product of η_t and η_g;
η_g	=	Generator efficiency;
η_t	=	Turbine efficiency;
γ	=	Specific weight of water;
ν	=	Kinematic Viscosity;
$\sum k_{1-2}$	=	Sum of local losses in penstock due to entrance, bends, pipe fittings and gates.

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